

Fake violations of the quantum Bell-parameter bound

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Shortcomings of experimental techniques are usually assumed to diminish nonclassical properties of quantum systems. Here it is demonstrated that this standard assumption is not true in general. It is theoretically shown that the inability to resolve different photon numbers in photodetection may pseudo-increase a measured Bell parameter. Under proper conditions one even pseudo-violates the quantum Cirel'son bound of the Bell parameter, the corresponding density operator fails to be positive semi-definite. This paradox can be resolved by appropriate squash models.

PACS numbers: 03.65.Ud, 42.50.Xa, 42.65.Lm

I. INTRODUCTION

Quantum mechanics is not a local realistic theory; this means that values of observables may not be pre-defined. This conclusion from the famous work by Einstein, Podolsky, and Rosen [1] still attracts a great deal of attention from physicists. Bell [2] has proposed a formal framework for this discussion. He has formulated inequalities that are valid for the local realism, but they are violated by quantum physics. According to this, the Bell parameter is less than the value of 2 for any local realistic theory. In the quantum world this parameter may exceed this threshold up to the level of $2\sqrt{2}$, known as the Cirel'son bound [3]. The increasing interest in this subject has also stimulated a variety of experiments, see e.g. [4].

An intriguing question is whether the Bell parameter may exceed the Cirel'son quantum bound. Popescu and Rohrlich have considered the consequences of two axioms: nonlocality and relativistic causality [5]. They have shown that as a consequence of the axioms, quantum mechanics appears as a particular representative of a more general theory. The latter includes the possibility of violating the Cirel'son quantum bound, with the maximum value of the Bell parameter being 4. Another situation has been considered by Cabello [6]. He has demonstrated that two qubits of a three-qubit system may also violate the Cirel'son quantum bound up to the level of 4 based on the standard quantum theory.

The shortcomings of experimental techniques, including losses, noise, etc., play two different roles in Bell-type experiments. First, they lead to a decrease of the Bell parameter; even for the Bell states the violation is no longer maximal (see, e.g., [7]). Second, small values of the detection efficiency are the subject of a loophole for local realism in Bell inequalities [8]. For a discussion of other loopholes we refer the reader to [9]. The mentioned

disadvantages also result in problems with the implementation of quantum-key distribution (QKD) protocols (cf., e.g., [10]).

A special example of such shortcomings is the impossibility to resolve between different numbers of photons. For on-off detectors it is only possible to distinguish between the presence and absence of detected photons. In addition, some standard experiments restrict the interpretation to low-dimensional Hilbert spaces, while the real electromagnetic field is characterized by an infinite-dimensional Hilbert space. In this case, the measurement procedure and further postprocessing in fact maps or squashes the higher-dimensional Hilbert space onto a low-dimensional one. If such a procedure is consistent, an appropriate squash model exists [11].

The aim of this paper is to show that the straightforward measurement procedure in Bell-type experiments may result in a fake enhancement of nonclassical properties. Moreover, we predict that even the limits of quantum physics may be pseudoviolated and that the Bell parameter exceeds the quantum Cirel'son bound. In such cases, the reconstructed density operator fails to be positive semidefinite. Of course, such fake effects do not mean that we predict violations of quantum physics. Nevertheless, the correct postprocessing, which is consistent with a properly defined squash model, resolves this problem and yields acceptable results. A clear understanding of this problem is of importance for the security of quantum communication.

Our paper is organized as follows. In Sec. II we consider the fake violation of the Cirel'son bound of the Bell parameter by using on-off detectors. A resolution of this paradoxical result by a proper squash model is given in Sec. III. An alternative way of demonstrating the fake violation of quantum physics under study by the reconstruction of ill-defined two-qubit density operators is considered in Sec. IV. In Sec. V we give a summary and some conclusions.

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II. BELL-TYPE EXPERIMENT

Let us start with a standard experimental setup briefly sketched in Fig. 1. The source of entangled photons irradiates into four modes: the horizontal and vertical polarized modes at site A and similar modes at site B . Typically each polarization analyzer consists of a half-wave plate, which can change the polarization direction to the angles θ_A and θ_B at A and B sites, respectively, a polarization beam splitter, and two detectors for the reflected and transmitted modes.

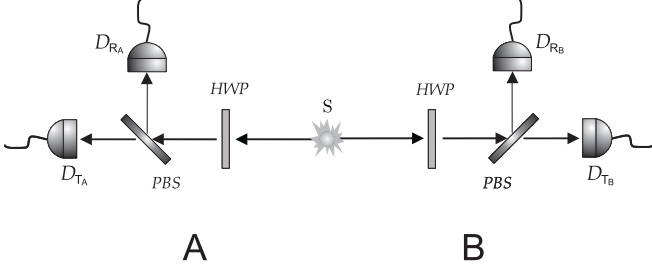


FIG. 1: A typical experimental setup for checking the violation of Bell inequalities. The source S produces entangled photon pairs. The polarization analyzers A and B consist of half-wave plates (HWP) polarizing beam splitters (PBS), and two pairs of detectors: D_{TA} (D_{TB}) for the transmitted signal and D_{RA} (D_{RB}) for the reflected signal at the A site (B site).

Let $P_{i_A, i_B}(\theta_A, \theta_B)$ be the probability to get a click in one of the detectors $i_A = \{T_A, R_A\}$ at site A and in one of the detectors $i_B = \{T_B, R_B\}$ at site B for the angles of the polarization analyzers θ_A and θ_B , respectively. The correlation coefficient is given by

$$E(\theta_A, \theta_B) = \frac{P_{\text{same}}(\theta_A, \theta_B) - P_{\text{different}}(\theta_A, \theta_B)}{P_{\text{same}}(\theta_A, \theta_B) + P_{\text{different}}(\theta_A, \theta_B)}, \quad (1)$$

where

$$P_{\text{same}}(\theta_A, \theta_B) = P_{T_A, T_B}(\theta_A, \theta_B) + P_{R_A, R_B}(\theta_A, \theta_B), \quad (2)$$

is the probability to get clicks on both detectors in the transmission channels or both detectors in the reflection channels, and

$$P_{\text{different}}(\theta_A, \theta_B) = P_{T_A, R_B}(\theta_A, \theta_B) + P_{R_A, T_B}(\theta_A, \theta_B) \quad (3)$$

is the probability to get clicks on the detectors in the transmission channel at one site and the reflection channel at another site. The Clauser-Horne-Shimony-Holt (CHSH) Bell-type inequality [12] states that the parameter

$$\mathcal{B} = \left| E(\theta_A^{(1)}, \theta_B^{(1)}) - E(\theta_A^{(1)}, \theta_B^{(2)}) \right| + \left| E(\theta_A^{(2)}, \theta_B^{(2)}) + E(\theta_A^{(2)}, \theta_B^{(1)}) \right|, \quad (4)$$

also referred to as the Bell parameter, cannot exceed the value of 2 for local-realistic theories. In quantum theory

it is bounded by the Cirel'son bound of $2\sqrt{2}$. As shown below, this bound may be violated in the presence of experimental imperfections.

According to the photodetection theory [13], the probability $P_{i_A, i_B}(\theta_A, \theta_B)$ for on-off detectors is given by

$$P_{i_A, i_B}(\theta_A, \theta_B) = \sum_{n, m=1}^{+\infty} \text{Tr} \left(\hat{\Pi}_{i_A}^{(n)} \hat{\Pi}_{i_B}^{(m)} \hat{\Pi}_{j_A}^{(0)} \hat{\Pi}_{j_B}^{(0)} \hat{\rho} \right), \quad (5)$$

$i_A \neq j_A$, $i_B \neq j_B$, where $\hat{\rho}$ is the density operator of the light at the input ports of the polarization analyzers and

$$\hat{\Pi}_{i_{A(B)}}^{(n)} =: \frac{(\eta \hat{n}_{i_{A(B)}} + N_{\text{nc}})^n}{n!} \exp(-\eta \hat{n}_{i_{A(B)}} - N_{\text{nc}}) : \quad (6)$$

is the positive operator-valued measure in the presence of non-unit efficiency η and the mean number of noise counts N_{nc} (originated from the internal dark counts and the background radiation), $::$ means normal ordering; see [14]. For simplicity we assume the detection efficiencies and noise-count rates to be equal for all detectors. The photon-number operator in the channel $i_{A(B)}$, $\hat{n}_{i_{A(B)}} = \hat{a}_{i_{A(B)}}^\dagger \hat{a}_{i_{A(B)}}$ can be expressed by the horizontal and vertical modes, $\hat{a}_{H_{A(B)}}$ and $\hat{a}_{V_{A(B)}}$, respectively, using the input-output relations for the polarization analyzers:

$$\hat{a}_{T_{A(B)}} = \hat{a}_{H_{A(B)}} \cos \theta_{A(B)} + \hat{a}_{V_{A(B)}} \sin \theta_{A(B)}, \quad (7)$$

$$\hat{a}_{R_{A(B)}} = -\hat{a}_{H_{A(B)}} \sin \theta_{A(B)} + \hat{a}_{V_{A(B)}} \cos \theta_{A(B)}. \quad (8)$$

In the case when the source S generates the perfect Bell state, the Bell parameter \mathcal{B} , calculated by using Eqs. (1)-(8), cannot exceed the Cirel'son bound of $2\sqrt{2}$. However, realistic sources produce, as a rule, more complicated states. Consider a parametric down-conversion (PDC) process of entangled-photon generation [9, 15–17]. Such a source, for example, has been recently used for transferring entanglement over a long-distance free-space channel [7]. The state $\hat{\rho} = |\Psi\rangle\langle\Psi|$, emitted by the PDC source, is of the form [15, 16]

$$|\Psi\rangle = (\cosh \chi)^{-2} \sum_{n=0}^{+\infty} \sqrt{n+1} \tanh^n \chi |\Phi_n\rangle, \quad (9)$$

where χ is the squeezing parameter and

$$|\Phi_n\rangle = \frac{1}{\sqrt{n+1}} \sum_{m=0}^n (-1)^m |n-m\rangle_{H_A} |m\rangle_{V_A} |m\rangle_{H_B} |n-m\rangle_{V_B}. \quad (10)$$

For small χ , one often restricts the consideration to $n=1$, representing a Bell state. Note that it was already mentioned in [9], that in order to close loopholes for local realism one should discriminate between photon numbers while using such sources.

An analysis of higher-term contributions in [16, 17] consists of the restriction to a few terms in the series (9).

However, since this state is Gaussian, we may obtain the exact analytical expression for Eq. (5) which enables us to provide a strict analysis of the problem, as done in Ref. [18]. For simplicity, we suppose that the losses for all four modes are equal. It is possible to show that in this case one can include them into the detection losses and describe all the losses by the efficiency η in Eq. (6).

After straightforward algebra (cf. Ref. [18]) by using the state (9) in Eq. (5), the probability $P_{i_A, i_B}(\theta_A, \theta_B)$ for the state (9) can be written as

$$P_{i_A, i_B}(\theta_A, \theta_B) = \eta^4 (1 - \tanh^2 \chi)^4 \exp(-4N_{nc}) \times \left[\frac{1}{C_0 + 2C_1 + C_{i_A, i_B}} - \frac{2e^{-N_{nc}}}{C_0 + C_1} + \frac{1}{C_0} \right]. \quad (11)$$

Here

$$C_0 = \left\{ \eta^2 \tanh^2 \chi - [1 + (\eta - 1) \tanh^2 \chi]^2 \right\}^2, \quad (12)$$

$$C_1 = \eta(1 - \eta)(1 - \tanh^2 \chi) \tanh^2 \chi \times \left\{ \eta^2 \tanh^2 \chi - [1 + (\eta - 1) \tanh^2 \chi]^2 \right\}, \quad (13)$$

$$C_{T_A, T_B} = C_{R_A, R_B} = \eta^2 \tanh^2 \chi (1 - \tanh^2 \chi)^2 \times \left[(1 - \eta)^2 \tanh^2 \chi - \sin^2(\theta_A - \theta_B) \right], \quad (14)$$

$$C_{T_A, R_B} = C_{R_A, T_B} = \eta^2 \tanh^2 \chi (1 - \tanh^2 \chi)^2 \times \left[(1 - \eta)^2 \tanh^2 \chi - \cos^2(\theta_A - \theta_B) \right]. \quad (15)$$

Now we may insert Eq. (11) in Eq. (1) and apply the result in Eq. (4) for the analysis of the Bell parameter.

In Fig. 2 we show with solid lines the dependence of the maximal value of the Bell parameter \mathcal{B} on the parameter $\tanh \chi$ for different values of the efficiency η and for $N_{nc} = 10^{-6}$. For small losses this parameter exceeds the Cirel'son bound of $2\sqrt{2}$, representing a fake violation of quantum physics. This originates from a straightforward but inconsistent postprocessing of the data [19].

The effect of fake violations of the quantum Bell-parameter bound does not occur when one uses photon-number-resolving detectors, for which Eq. (5) for the probability $P_{i_A, i_B}(\theta_A, \theta_B)$ should be rewritten as

$$P_{i_A, i_B}(\theta_A, \theta_B) = \text{Tr} \left(\hat{\Pi}_{i_A}^{(1)} \hat{\Pi}_{i_B}^{(1)} \hat{\Pi}_{j_A}^{(0)} \hat{\Pi}_{j_B}^{(0)} \hat{\rho} \right). \quad (16)$$

In this case, Eq. (11) is replaced by

$$P_{i_A, i_B}(\theta_A, \theta_B) = \eta^4 (1 - \tanh^2 \chi)^4 \times \exp(-4N_{nc}) \left[2 \frac{C_{i_A} C_{i_B}}{C_0^3} - \frac{C_{i_A, i_B}}{C_0^2} + N_{nc} \frac{C_{i_B}}{C_0^2} + N_{nc} \frac{C_{i_A}}{C_0^2} + N_{nc}^2 \frac{1}{C_0} \right]. \quad (17)$$

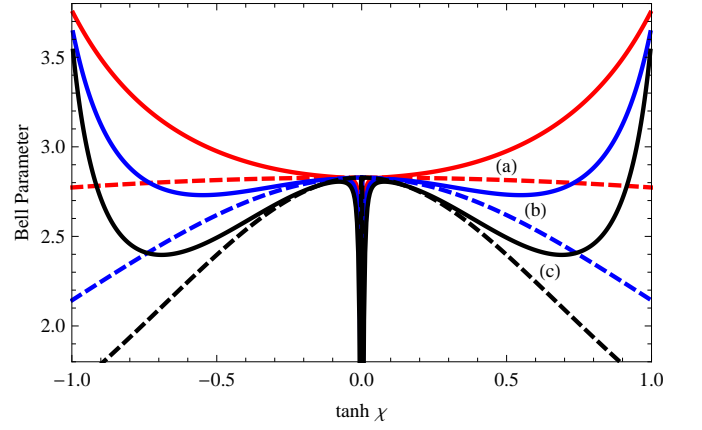


FIG. 2: (Color online) Maximal value of the Bell parameter \mathcal{B} , obtained for on-off (solid lines) and photon-number-resolving (dashed lines) detectors, vs the parameter $\tanh \chi$ for the mean value of noise counts $N_{nc} = 10^{-6}$ and the efficiencies (a) $\eta = 0.9$, (b) $\eta = 0.6$, and (c) $\eta = 0.4$. The minimum at $\tanh \chi = 0$ is caused by noise counts.

In addition, results obtained with photon-number-resolving detectors demonstrate, as a rule, smaller values of the Bell parameter compared with on-off detectors; see Fig. 2 and Ref. [18] for details of calculations.

It is worth noting that photocounting with the resolution of photon numbers is approximately possible by splitting an initial light beam into a large number of low-intensity beams [20]. Detection of photons in each beam by an array of photodiodes allows one to get more insight into the number statistics of the detected photons. Similarly, one can use time multiplexing in a fiber loop with one photodiode [21]. However, such measurement techniques only partly allow one to infer the photon number statistics. The question remains whether or not such detection schemes are suited to completely eliminate the fake effects under study. This would require a more detailed analysis, which is beyond the scope of the present paper.

III. DOUBLE-CLICK EVENTS

In the following we will show that one can overcome the violation of the Cirel'son bound, even when using on-off detectors. A consistent result can be obtained by the application of a proper squash model [11]. The point is that the straightforward scheme considered above discards so-called double-click events when both detectors at the receiver station $A(B)$ click [19].

Let us assign to the double-click events random bits with a probability $1/2$. This changes the situation completely. In this case, Eq. (5) for the probability

$P_{i_A, i_B}(\theta_A, \theta_B)$ is replaced by

$$\begin{aligned}
P_{i_A, i_B}(\theta_A, \theta_B) = & \sum_{n,m=1}^{+\infty} \text{Tr} \left(\hat{\Pi}_{i_A}^{(n)} \hat{\Pi}_{i_B}^{(m)} \hat{\Pi}_{j_A}^{(0)} \hat{\Pi}_{j_B}^{(0)} \hat{\rho} \right) \\
& + \frac{1}{2} \sum_{n,m,k=1}^{+\infty} \text{Tr} \left(\hat{\Pi}_{i_A}^{(n)} \hat{\Pi}_{i_B}^{(m)} \hat{\Pi}_{j_A}^{(k)} \hat{\Pi}_{j_B}^{(0)} \hat{\rho} \right) \\
& + \frac{1}{2} \sum_{n,m,k=1}^{+\infty} \text{Tr} \left(\hat{\Pi}_{i_A}^{(n)} \hat{\Pi}_{i_B}^{(m)} \hat{\Pi}_{j_A}^{(0)} \hat{\Pi}_{j_B}^{(k)} \hat{\rho} \right) \\
& + \frac{1}{4} \sum_{n,m,k,l=1}^{+\infty} \text{Tr} \left(\hat{\Pi}_{i_A}^{(n)} \hat{\Pi}_{i_B}^{(m)} \hat{\Pi}_{j_A}^{(k)} \hat{\Pi}_{j_B}^{(l)} \hat{\rho} \right). \quad (18)
\end{aligned}$$

For the sake of simplicity, in the following, we only consider the most critical case of lossless detectors, $\eta = 1$, with no noise counts, $N_{nc} = 0$. Under these conditions the violation of the quantum Cirel'son bound, as considered in the previous section, attains its maximum. Substituting state (9) into Eqs. (1) and (18), one gets

$$E(\theta_A, \theta_B) = -\frac{\cos[2(\theta_A - \theta_B)]}{D}, \quad (19)$$

where

$$\begin{aligned}
D = & 1 - \frac{1}{2} \tanh^2 \chi \sin^2[2(\theta_A - \theta_B)] \\
& + \frac{9 + 3(1 - \tanh^2 \chi)}{2 \tanh^2 \chi (1 - \tanh^2 \chi)^2} \\
& \times \left\{ 1 - \tanh^2 \chi + \frac{1}{4} \tanh^4 \chi \sin^2[2(\theta_A - \theta_B)] \right\} \\
& + \frac{[1 - 2(1 - \tanh^2 \chi)^2](2 - \tanh^2 \chi)}{\tanh^2 \chi (1 - \tanh^2 \chi)^2}. \quad (20)
\end{aligned}$$

By applying the squash model in this way, the maximal value of the Bell parameter no longer exceeds the Cirel'son bound of $2\sqrt{2}$, see. Fig. 3. This result demonstrates the importance of consistent squash models. In particular, it is necessary to include the double-click events into the data postprocessing.

IV. TWO-QUBIT DENSITY OPERATOR

In order to demonstrate the inconsistency of the straightforward postprocessing when double-click events are ignored, we provide the following argument. Any density operator $\hat{\rho}$ of a two-qubit system, with zero mean values of all spin projections, can be expanded into a nonorthogonal and linearly independent basis as

$$\hat{\rho} = \frac{\hat{I} \otimes \hat{I}}{4} + \frac{1}{4} \sum_{i,j=1}^3 E(\theta_A^{(i)}, \varphi_A^{(i)}; \theta_B^{(j)}, \varphi_B^{(j)}) \hat{\Xi}_A^{(i)} \otimes \hat{\Xi}_B^{(j)}, \quad (21)$$

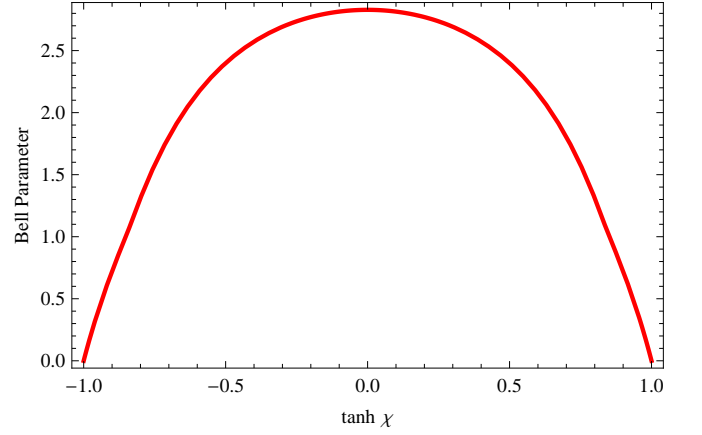


FIG. 3: (Color online) Maximal value of the Bell parameter B , obtained for lossless on-off detectors without noise counts.

where \hat{I} is the 2×2 identity matrix. E denotes the correlation coefficients for spin projections with the Euler angles $\theta_{A(B)}^{(i)}, \varphi_{A(B)}^{(i)}$ ($i = 1, 2, 3$). The angles $\varphi_{A(B)}^{(i)}$ have been introduced to obtain a linear-independent basis of the matrix

$$\hat{\Xi}_{A(B)}^{(i)} = \sum_{k=1}^3 g^{(k,i)} \begin{pmatrix} \cos 2\theta_{A(B)}^{(k)} & e^{-i\varphi_{A(B)}^{(k)}} \sin 2\theta_{A(B)}^{(k)} \\ e^{i\varphi_{A(B)}^{(k)}} \sin 2\theta_{A(B)}^{(k)} & -\cos 2\theta_{A(B)}^{(k)} \end{pmatrix}, \quad (22)$$

where $g^{(k,i)}$ is the metric tensor, which is inverse to

$$\begin{aligned}
(g^{-1})_{(k,i)} = & \cos 2\theta_{A(B)}^{(i)} \cos 2\theta_{A(B)}^{(k)} \\
& + \sin 2\theta_{A(B)}^{(i)} \sin 2\theta_{A(B)}^{(k)} \cos(\varphi_{A(B)}^{(i)} - \varphi_{A(B)}^{(k)}). \quad (23)
\end{aligned}$$

The above-discussed mapping onto a two-qubit state implies the identification of the coefficients $E(\theta_A^{(i)}, \varphi_A^{(i)}; \theta_B^{(j)}, \varphi_B^{(j)})$ in Eq. (21) with relation (1), as is usually done in experiments. To include the dependencies on the angles $\varphi_{A(B)}^{(i)}$ in the experimental setup (see Fig. 1), one should complete it with phase shifters for the horizontal polarized modes. In this case one must replace $\hat{a}_{H_{A(B)}}$ with $\hat{a}_{H_{A(B)}} \exp(i\varphi_{A(B)}^{(i)})$ in Eqs. (7) and (8). In the case of using on-off detectors without considering double-click events, Eq. (21) no longer represents a correctly defined density operator for a two-qubit system. First, it is not uniquely defined since it depends on the chosen angles $\theta_{A(B)}^{(i)}$ and $\varphi_{A(B)}^{(i)}$, which represent different choices of the basis. Second and most importantly, for some values of the angles, operator (21) appears not to be positive semidefinite, which contradicts the fundamental properties of a quantum state.

In Fig. 4 we show the minimum eigenvalue of the two-qubit density operator $\hat{\rho}$ inferred with on-off detectors without considering double-click events. It is clearly seen that for different choices of the angles the minimum eigenvalue of $\hat{\rho}$ can be either non-negative or it may become negative. In one of the cases, the mapping onto

a two-qubit system clearly fails: the associated effective state does not show the properties of a correctly defined quantum state. As a consequence, the Bell parameter (4) may even exceed the quantum Cirel'son bound of $2\sqrt{2}$. Equation (21) can be used for reconstructing a two-qubit density operator by using the measured correlation coefficients. If the result of this reconstruction is not positive semidefinite, it fails to describe a quantum state. This is an alternative possibility to demonstrate the fake violation of quantum physics.

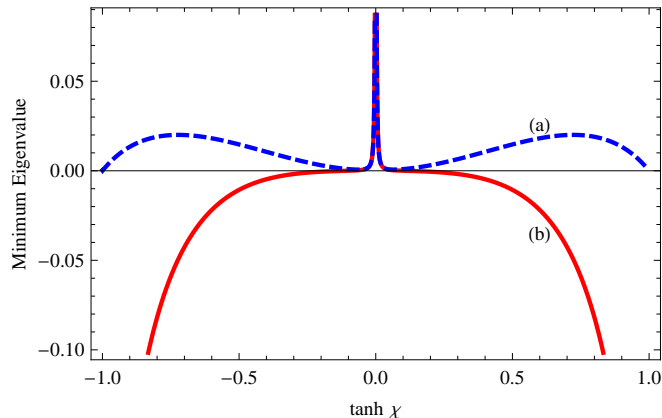


FIG. 4: (Color online) The minimum eigenvalue of the density operator $\hat{\rho}$ [see Eq. (21)] vs the parameter $\tanh \chi$ for the mean value of noise counts $N_{nc} = 10^{-6}$, the efficiency $\eta = 0.6$, and the following sets of angles: (a) $\theta_A^{(1)} = \theta_B^{(1)} = \pi/4$, $\varphi_A^{(1)} = \varphi_B^{(1)} = 0$, $\theta_A^{(2)} = \theta_B^{(2)} = \pi/4$, $\varphi_A^{(2)} = \varphi_B^{(2)} = \pi/2$, $\theta_A^{(3)} = \theta_B^{(3)} = 0$, $\varphi_A^{(3)} = \varphi_B^{(3)} = 0$; (b) $\theta_A^{(1)} = \pi/8$, $\varphi_A^{(1)} = 0$, $\theta_A^{(2)} = 9\pi/4$, $\varphi_A^{(2)} = \pi/2$, $\theta_A^{(3)} = \pi$, $\varphi_A^{(3)} = 0$, $\theta_B^{(1)} = 3\pi/15$, $\varphi_B^{(1)} = 0$, $\theta_B^{(2)} = -\pi/24$, $\varphi_B^{(2)} = \pi/2$, $\theta_B^{(3)} = \pi$, $\varphi_B^{(3)} = 0$.

V. SUMMARY AND CONCLUSIONS

Our analysis of biphoton Bell-type experiments has demonstrated that the impossibility of discriminating between different photon numbers is a kind of imperfection leading to surprising results. First, it leads to a pseudo-increase of the measured Bell parameter. Second, we also predict that for high detection efficiency the measured Bell parameter can even show a pseudo-violation of the fundamental limit of quantum physics, the Cirel'son bound.

The reason for such fake violations is shown to consist in mappings of a continuous-variable quantum state onto an ill-defined two-qubit state that fails to be positive semidefinite. Such fake effects disappear when one includes in the consideration a consistent postprocessing of the measured data by a proper squash model. The knowledge of such effects plays an important role in the analysis of some quantum-key-distribution protocols.

Acknowledgments

The authors gratefully acknowledge support by the Deutsche Forschungsgemeinschaft through SFB 652. We also strongly appreciate useful discussions with Hoi-Kwong Lo.

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